paper (Bhat, Chakraverty and Stiharu), the recurrence scheme used in references [3–11] makes use of the two latest generated polynomials in generating a new one.

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AUTHORS' REPLY

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The authors would like to thank Dr Xu and Dr Jiang for their nice comments on our Letter to the Editor [1].

The idea of constructing orthogonal polynomials, satisfying the boundary conditions of vibrating structures, using the Gram–Schmidt orthogonalization process, was originally proposed by Bhat [2] in 1985. These were used to study the vibration of one-dimensional structures or vibration of rectangular plates where variables can be separated. Bhat's method was used to study plate problems with complicating effects for a Ph.D. thesis by Kim [3], under the supervision of Professor S.M. Dickinson, and resulted in several publications [4–6]. Based on an article by Tom Koornwinder in a Monograph edited by Askey [7], two-dimensional orthogonal polynomials satisfying the boundary conditions were constructed by Bhat [8] in 1987 for the first time, to study the vibration of traingular plates by the Rayleigh–Ritz method. It was also mentioned that the method could be used to study polygonal plates as well. As stated in that paper, the mth orthogonal polynomial had to be orthogonalized with respect to all the $(m-1)$ orthogonal polynomials constructed so far. This method was used by Lam, Liew and co-workers later [10–12]. The method was also used by Dr Chakraverty [13], supervised by Professor B. Singh, who published several studies subsequently [14–16]. Liew and Lam's studies were essentially following the same technique of orthogonalization as proposed by Bhat [8], i.e., mth polynomial being orthogonalized with respect to all the previous orthogonal polynomials. In such a "recurrence relation", the number of orthogonalizations increased with the construction of each higher member of the set.

Recently, the authors came across the two papers by Kowalski [17, 18], which provided a more efficient recurrence relation between three successive "classes" of polynomials in the sequence. Accordingly reference [1] was formulated and this type of recurrence relation to construct two-dimensional boundary characteristic orthogonal polynomials has been used for the first time. Here it is not necessary to orthogonolize the mth orthogonal polynomial with respect to all the $(m-1)$ orthogonal polynomials constructed so far. For example, referring to the pyramid scheme shown in Figure 1, it is necessary to orthogonalize the previously constructed members in the same class and also two classes below it. This reduces the number of orthogonalization operations as one constructs more number of orthogonal polynomials. In hindsight, the title of reference [1] could have been, "An efficient recurrence relation scheme for the generation of the two-dimensional boundary characteristic orthogonal polynomials to study vibration of plates".

As an additional note, the Boundary Characteristic Orthogonal Polynomials as proposed by Bhat [2, 8] have been used in more than 100 publications by researchers around the world and have been the subject of four Ph.D. theses for

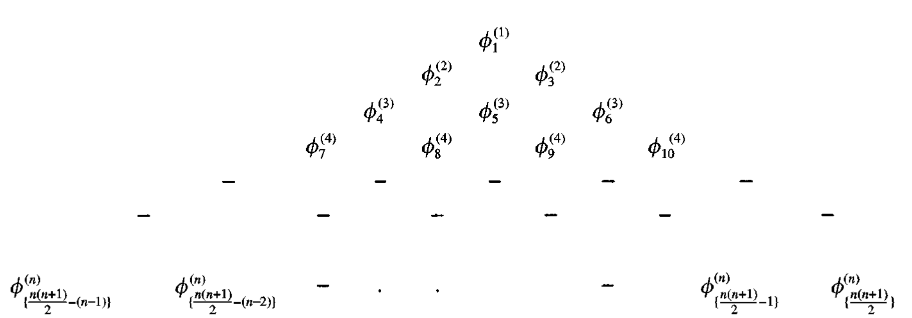

Figure 1. Pyramid scheme for efficient orthogonalization of two-dimensional polynomials.

various applications. A review paper giving all these references will appear in the Shock and Vibration Journal [19].

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